During the past few weeks we have revisited the characteristics of exponential equations in many different contexts. We have recalled the facts that if $b > 1$ in an equation in the form $y = a(b)^x$, then the equation represents exponential growth and if $0 < b < 1$, the same equation will represent exponential decay. In class one day, a student raised the question "What if the base is negative?" As a result of this question we investigated what would happen in an equation such as $y = 5(-2)^x$, for example. Students entered this equation into y = in their calculators, set the TblStart at 0 and the ΔTbl at 1, looked at the table, and quickly realized the pattern that results. One student, however, had the table on her calculator set so that TblStart was 0 and ΔTbl was 0.5. She noticed that every other entry in the table said ERROR and wondered why.

Analyze this situation and summarize your work below. Include the conclusions that you reached as a result of your investigation. Keep in mind that you can investigate beyond the confines of the situation described above.

During the past few weeks we have revisited the characteristics of exponential equations in many different contexts. We have recalled the facts that if $b > 1$ in an equation in the form $y = a(b)^x$, then the equation represents exponential growth and if $0 < b < 1$, the same equation will represent exponential decay. In class one day, a student raised the question "What if the base is negative?" As a result of this question we investigated what would happen in an equation such as $y = 5(-2)^x$, for example. Students entered this equation into y = in their calculators, set the TblStart at 0 and the ΔTbl at 1, looked at the table, and quickly realized the pattern that results. One student, however, had the table on her calculator set so that TblStart was 0 and ΔTbl was 0.5. She noticed that every other entry in the table said ERROR and wondered why.

Analyze this situation and summarize your work below. Include the conclusions that you reached as a result of your investigation. Keep in mind that you can investigate beyond the confines of the situation described above.

There is a logical explanation for the errors in every other entry. The error occurs every time x = .5, 1.5, 2.5, etc (every consecutive .5 value). .5 is the same as the fraction $\frac{1}{2}$. So, if you plug $\frac{1}{2}$ in for the exponent for a negative number (in this case 5(-2)) Then using the rule about expressing a term w/ a fraction exponent in radical form, this cannot be done. This is because $\frac{1}{2}$ is another way of meaning a square root of a number. It is not possible to have a square root of a negative number, because a negative times a negative = positive, and a positive × positive = positive. So, -2 doesn't have a square root, making an error on the calculator. It is the same with $y=5(-2)^x$ with x = 8.5, because it would still be -2 to the $\frac{17}{2}$ power, meaning one would need the $\sqrt{-2}$. If x were to increase by .3, every odd decimal would result in an error for y-value. This is because $.3 = \frac{3}{10}$, so you would be finding $\sqrt[10]{-2}$. This is impossible, for any number multiplied by itself an even number of times will wind up being even. The reason why $(-2)^{.6}$ works is because $.6 = 3/5$, meaning you are taking the $\sqrt[5]{-2}$. This is defined because a negative number × itself an odd number of times is a negative number.

During the past few weeks we have revisited the characteristics of exponential equations in many different contexts. We have recalled the facts that if $b > 1$ in an equation in the form $y = a(b)^x$, then the equation represents exponential growth and if $0 < b < 1$, the same equation will represent exponential decay. In class one day, a student raised the question "What if the base is negative?" As a result of this question we investigated what would happen in an equation such as $y = 5(-2)^x$, for example. Students entered this equation into y = in their calculators, set the TblStart at 0 and the ΔTbl at 1, looked at the table, and quickly realized the pattern that results. One student, however, had the table on her calculator set so that TblStart was 0 and ΔTbl was 0.5. She noticed that every other entry in the table said ERROR and wondered why.

Analyze this situation and summarize your work below. Include the conclusions that you reached as a result of your investigation. Keep in mind that you can investigate beyond the confines of the situation described above.

The pattern when Δtbl is 1 is that each y-value is multiplied by b to get the next y-value which means that y will alternate between positive and negative. There is an error for when it is an integer and a half for a value of x because when written as a fractional power, the denominator will be 2. This can be rewritten as the square root of b to the power of the numerator. Since b is negative, $\sqrt{b}$ is unreal. For example, if x is 1.5, or $\frac{3}{2}$ in $y = 5(-2)^x$ the equation would be $y = 5(-2)^{\frac{3}{2}}$, which is the same as $y = 5(\sqrt{-2})^3$. $\sqrt{-2}$ is not a real number.

This does not mean that the graph is just defined for integers. It is defined for all fractions with an odd denominator in simpilest form. Odd denominators result in odd roots, which are defined for negative numbers. For example, if Δtbl is set to 1/3, all entries are defined. If x is 1.6, or $\frac{5}{3}$, $y = 5(\sqrt{-2})^x$ becc $y = 5(-2)^{\frac{5}{3}}$ or $5(\sqrt[3]{-2})^5$. The cube root of -2 is -1.2599, to the power of 5 is -3.1748, times 5 is -15.8740, which is a real number. All values would be defined if Δtbl was set to $\frac{1}{5}$ $\frac{1}{7}$ $\frac{1}{9}$ as well, because 5th, 7th, 9th etc. roots are defined for negative numbers.